Influence of Converter-Connected Distributed Generation on Distribution Network Losses

Lieven Degroote, Bert Renders, Bart Meersman and Lieven Vandevelde

Electrical Energy Laboratory (EELAB),
Department of Electrical Energy, Systems and Automation (EESA), Ghent University, Gent, Belgium.
Email: Lieven.Degroote@UGent.be

Abstract Distributed generation (DG) is predicted to play an increasing role in the electric power system of the near future. There are different types of DG units from the constructional and technological points of view, which have a different influence on the distribution network. The impact of DG on the network is considered, with a more extensive discussion on the influence of DG on the network losses. The calculation of these network losses is applied in several optimisation programs to quantify an optimal size and location of the DG unit. In these programs the power quality can be taken into account but harmonic losses are rarely considered, although it is known that several DG topologies can have a negative influence on the total harmonic distortion and thus on the harmonic losses. Therefore, this paper will investigate the influence of converter-connected DG units on fundamental and harmonic losses in the distribution network.

Key words

Harmonics, losses, radial distribution networks, distributed generation

1. Introduction

Electrical power systems are nowadays evolving from centralised systems, where generator plants are connected to the transmission networks, to a decentralized system, with smaller generation units connected directly to the distribution networks and thus near consumption. The main incentives for this evolution are the environmental concerns (use of renewable energy, use of combined heat and power) and the security of supply (diversity of energy sources, markets with large number of actors). The injections of power by the distributed generators may change the magnitude and even the direction of the power flow in the network. This has several implications for the operation and planning of the network and has several technical and economic consequences [1–17]. In this framework it is clear that DG units affect the network losses, which are defined as the difference between the energy sent out from the generating stations and the energy metered at the customer premises. Losses occur in all systems of electricity transmission and distribution. These are usually divided into two categories: technical (related to the characteristics of the carrier equipment, supply and demand patterns) and commercial (energy not accounted for, for example, theft and meter errors) [16, 18].

There are different types of DG units from the constructional and technological points of view [10], which have a different influence on the distribution network. The impacts of DG on the network are given in the next section, with a special emphasis on the influence of DG on the network losses [11, 15, 16]. The calculation of these network losses are applied in several optimisation programs to quantify an optimal size and location of the DG unit [2,6,13,14,17]. In these programs the power quality can be taken into account. Harmonic losses are rarely considered, although it is known that several DG topologies can have a negative influence on the total harmonic distortion [7] and thus on the harmonic losses. Therefore, this paper will investigate the influence of converter-connected DG units on harmonic losses in the distribution network. Some typical harmonic spectra of converter-connected photovoltaic (PV) systems will be applied [19] to perform the simulations.

2. Impact of DG on the distribution networks - an overview

The overall impact of DG units on the distribution network is discussed in [1,3,5,9,12]. The main topics are discussed below.

An extensive approach of the impact is presented in [1]. It deals with the influence of DG units of limited size (10 MW or less) on the power quality, reduction of losses and reliability. There are several advantages when using DG. However some drawbacks need to be considered as well. For instance, voltage regulators can be disturbed when the DG unit is connected closely to the regulator, because there is less power through the regulator. A rule of thumb is that DG units sustain the voltage profile if the current injected from the DG unit is less than 5% of the feeder load. The influence on flicker has also been described in [1], which is the consequence of variations in the DG output power. Other effects of DG units are the injections of harmonic currents with a difference between invertor-connected DG units and generator connected DG units. Furthermore impact on short circuit levels and the most importants islanding problems are discussed.

In [3] the emphasis lies more on the potential cost savings of DG and the influence of grid support (reduction in grid losses, typically saving of $10 - 15\%$, voltage support, and power factor correction) from an economical point of view. The main conclusions from simulations state that the effect of DG on the distribution network is largely dependent on the power flow in the network, that losses are reduced by the implementation of DG and that the network reliability can be affected by the increasing presence of DG.

Research performed in [5] tackles the problems introduced by Domestic Combined Heat and Power (DCHP), the authors state that the main problems will be commercial, such as loss of distribution use of system revenue, and regulatory.

In near urban or rural networks, problems with the voltage profile are considered to be the most important [9]. In urban networks the line current is observed to be the biggest problem. The authors state that DG can have a positive influence on the harmonics in the distribution network (in cases where synchronous machines absorb the harmonics).

The research in [12] introduces many indices to quantify the DG benefits. The most important ones are the reduced line and transformer losses, reduced central generating station reserve requirements, improved system voltage profile, increased system reliability and enhanced power quality, peak shaving, reduced environmental impacts, and relieved transmission and distribution congestion.

The above mentioned impacts are applied in several optimisation programs to determine an optimal size and siting of a DG unit [2, 6, 13, 14, 17]. The main target of these programs is to minimize the network losses while the cost of the DG unit is kept as low as possible.

The economic aspect is the main incentive to keep the losses in power systems low. The influence of DG units on the losses in power systems is discussed in [11, 15, 16].

The research performed in [11] states that both active power losses and reactive power consumption are reduced with small amounts of DG connected to the power system. But when penetration increases, the power losses may increase again.

In [15] an approach to compute annual loss variations is presented when different penetration and concentration levels of DG are connected to a distribution network. The impact on losses of different primary drivers for DG units, such as combined heat and power, wind power, photovoltaics, and fuel-cells, is analysed. Different types of load flow algorithms have been implemented to calculate the losses. In conclusion, the radial load flow algorithm is preferred in the case of larger networks. The first result that claims attention is the shape of all traces. Losses start to decrease when connecting small amounts of DG until they reach their minimum level. Once this minimum level is reached and DG penetration is still increased, losses marginally increase too. If DG penetration levels increase even further, the losses can be even higher than without DG connected. Regarding the impact of each type of primary driver, it can be observed that wind turbines have the least positive impact on losses, because the injected energy is intermittent, presenting high time variability, and does not match well with the feeder load pattern. The impact on losses of the rest of the technologies is also explained by the expected matching between hourly energy generation and hourly load patterns. The control of reactive power supplied or consumed by DG also impacts the energy losses. The authors state that only large DG generators should control voltages in real time. Medium and small generators can keep a constant power factor with time discretisation. The above mentioned conclusions can also been deduced from [16], but the authors remark that especially in rural networks the losses can increase for higher penetration levels in contradiction to urban and mixed networks the overall losses are reduced with DG presence.

In [4] the impact of DG on the power system transient stability has been investigated. The authors have concluded that the impact of DG on the power system transient stability depends both on the penetration level and the technology of the distributed generators. Unlike asynchronous generators, synchronous generators have more influence on transient stability, they decrease the overspeeding of the large scale generators, but they also decrease the transient stability by increasing the oscillation duration. DG based on power electronics decrease the overspeeding of generators, because it is disconnected during a fault.

The research presented in [8] presents an algorithm for Volt/VAr control in distribution networks with DG. Simulations have been performed in order to reveal that by proper placement of DG units and using appropriate controller for them, it is possible to have much better control for Volt/VAr in distribution networks with decreasing the system losses in the network.

3. Influence of converter connected DG on the distribution network losses

In the previous section the most important influences of DG on the distribution network have been discussed. It is clear that the reduction of network losses are of great interest in algorithms to determine the optimal size and placement of a DG unit. This, together with the power quality issues raised by the use of DG, are the most important incentives to investigate the influence of DG units on the harmonic losses in distribution networks and to examine the relative contribution of the harmonic losses to the total losses.

A. Distribution Network Model

The simulations that are applied to determine the influence of converter-connected DG on the total losses (fundamental and harmonic losses), are performed on a radial network. The topology of the radial network (Fig. 1) used in the simulations is based on the IEEE 13 node test feeder [20]. Every line section consists of four wires (three phase and neutral). The grey lines are cables while the black lines are overhead lines. The properties of these lines are given in Table 1.

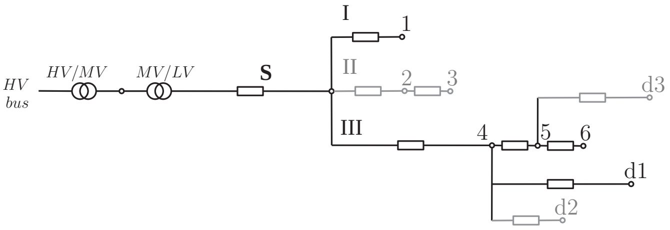

Figure 1: Topology of the considered radial distribution network.

Table 1: The line properties

Feeder	R (Ω/km)	L (mH/km)	C (μF/km)
S	0.0439	0.0573	/
I	0.351	0.344	/
II	0.226	0.223	0.29
III	0.0585	0.086	/
d1	0.351	0.344	/
d2	0.226	0.223	0.29
d3	0.226	0.223	0.29

B. Simulation Method

The network used for this simulation (Fig. 1) is implemented in symmetrical components. The iterative forward/backward sweep-based harmonic analysis is used for the simulations [21]. The forward/backward sweep technique can be divided into two parts:

1. *Backward Current Sweep:* For a radial feeder, the branch current can be calculated by summing the injection currents from the receiving bus towards the sending bus of the feeder.
2. *Forward Voltage Sweep:* Obviously, for a radial distribution system if branch currents were calculated, the bus voltages can be calculated from the sending bus toward the receiving bus of the feeder.

This process goes on until convergence is reached considering the harmonics till 2 kHz (40^{th} harmonic).

The branch current consists of the different currents injected in the nodes and the magnetizing currents of the transformers. These node currents originate from the linear and nonlinear loads present in the node or the capacitance in case of a cable segment.

The linear loads are modelled as an RL-impedance. Based on the fundamental direct component of the voltage at the specific node, the impedance is calculated to represent a constant power load. Mathematical instability can occur, because an iterative method is used. To avoid this problem the eigenvalues of the matrix that gives the relationship between the currents and the node voltages in iteration k and $k + 1$ have to be located in the unity circle. In general the magnitude of the load impedance and the impedance between the node and the sending bus of the feeder determine the magnitude of the eigenvalues. A higher load impedance has a positive effect on the stability, where a higher line impedance has a negative effect. Therefore, a reactance pair [22] has been inserted at every node. This pair consists of opposite reactances. The voltage used to calculate the current drawn by the linear load is not the node voltage but the voltage between the reactance pair, so physically the network does not change but from a mathematical point of view, the load impedance is higher and the line impedance has a lower absolute value.

The nonlinear loads used in the simulation presented in this paper are considered to be fixed harmonic current sources. To some nodes single or three phase currents sources will be connected to represent a large number of computer loads. Typical values of these currents are presented in [23]. Another source of harmonics in these simulations are the converter-connected units. These units are single-phase and the power injected into the network is fixed. In every iteration the fundamental component of the current is dependent on the voltage in the specific node. The harmonic currents, that are defined as fixed percentages of the fundamental current, are thus also dependent on the voltage in the node. Typical values for these harmonic currents can be found in [19]. Several output profiles are presented, for different percentages of output power referred to the nominal power. The simulations performed in this paper use the harmonic output spectrum for output powers equal or higher than the nominal power, which contains in terms of percentage the least harmonics.

The currents flowing in the capacitance of the cable are calculated analogously as the linear loads. The main problem for the capacitive current calculation is the decreasing impedance for higher harmonics, while the line impedance becomes larger with increasing harmonic order. The solution for this problem is the limitation of the increase of the capacitive current between two iterations. The current injected in the node lies between the current of the previous iteration and the calculated current. This has as consequence for the iterative procedure that not only the voltage has to be converged but also the currents drawn by the capacitors.

The transformers present in the network are not considered as linear elements but the magnetizing current (including harmonics) is calculated depending on the voltage at the primary and the secondary side. In every backward current sweep the magnetizing current is injected between the primary and the secondary of the transformers.

When the model has converged, the currents in the branches are used to calculate the fundamental and harmonic Joule losses in every feeder. The losses are calculated separately for every phase and the neutral conductor, the phase currents are determined using symmetrical components and the neutral current equals the homopolar component. In the calculations of the harmonic Joule losses the skin effect has been taken into account according to [24].

C. Simulation results

In order to investigate the influence of converter-connected PV-units on the losses, several simulations have been executed on the network presented in Fig. 1. In a first base case, no PV-units where connected on the network, the load distribution is given in Table 2.

Table 2: The load distribution

node	RL-load (kVA)	$\cos(\phi)$	computer load (kVA)	phase
1	20	0.9	6	A-B-C
2	15	0.95	5	A-B-C
3	7	0.9	/	/
4	4	0.9	/	/
5	7	0.95	3	A
6	3	0.85	1	A-B-C
d1	2	0.9	2	A-B-C
d2	6	0.9	/	/
d3	2	0.9	3	A-B-C

Table 3: The base case

node	v_1 (pu)	$\mathrm{THD_{V_A}}$ (%)	$\mathrm{THD_{V_B}}$ (%)	$\mathrm{THD_{V_C}}$ (%)
1	0.973	2.142	1.123	1.152
2	0.976	1.908	0.915	0.929
3	0.975	1.909	0.915	0.928
4	0.974	3.213	1.092	1.097
5	0.973	3.431	1.148	1.156
6	0.973	3.441	1.154	1.162

The absolute value of the direct voltage component and the Total Harmonic Voltage Distortion ($\mathrm{THD_V}$) for the base case are given in Table 3.

The influence of converter-connected distributed generation on the losses is revealed by considering fifteen different cases. The cases all have the same load distribution as presented in Table 2 but in one or several nodes a converter-connected distributed generator is connected as shown in Table 4. The DG can be connected in the following nodes: 1, 4, 6 and/or d2.

The results of the simulations are given in Fig. 2-5. Per case, four bars are depicted. The first three bars give the total losses for the the phases A, B and C respectively, where the fourth bar represents the losses in the neutral conductor. The losses are subdivided in two parts, the grey part represents the losses caused by the fundamental component of the current and the black part originates from the harmonic currents.

Table 4: Test cases

Case	1 kW-phase	4 kW-phase	6 kW-phase	d2 kW-phase
2	3-A			
3	9-A			
4	15-A			
5				3-A
6				9-A
7				15-A
8	3-A			3-A
9		3-A		
10		9-A		
11			3-A	
12			9-A	
13			3-B	
14			3-C	
15	3-A	3-A	3-A	3-A

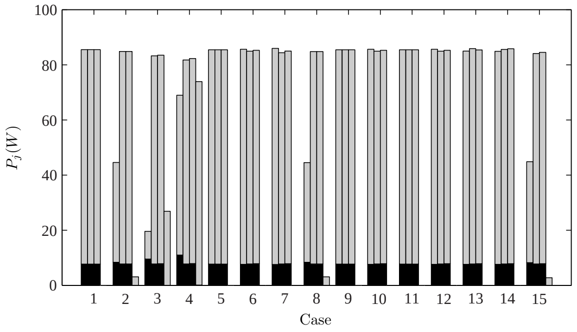

Figure 2: The losses in Feeder **I**.

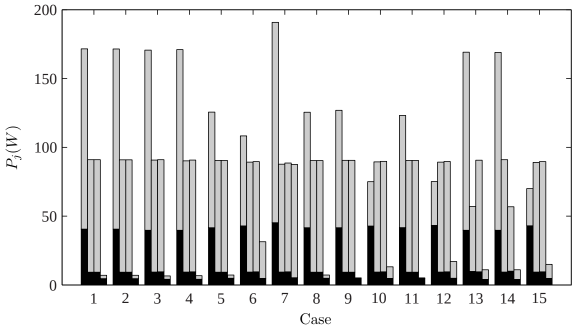

Figure 3: The losses in Feeder **III** and subfeeders.

In Fig. 2 the losses in Feeder **I** are given, Fig. 3 presents the sum of the losses in Feeder **III** and the subfeeders d1, d2 and d3. The losses in the supply feeder **S** are shown in Fig. 4. At last, Fig 5 gives an overview of the losses in the entire network.

The Joule losses in the base case amount to 2.75% of the power demand on the network. The harmonic losses add up to 11.45% of the fundamental losses. The main losses are situated in the supply feeder that transports all the power from the central generation. The harmonic losses in the supply feeder run up to 7.72% for the phases B and C, but to 15.16% in phase A, due to the nonlinear asymmetrical load in node 5. The losses in the neutral conductor (unbalance) account in this base case for 0.50% of the total losses in the entire network.

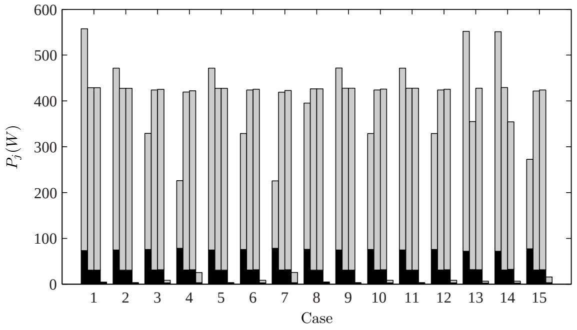

Figure 4: The losses in Feeder **S**.

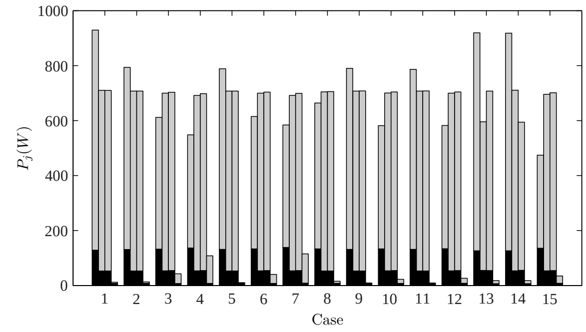

Figure 5: The losses in the entire network.

1) *Influence of increased input power*

In cases 2-4 the power inserted in node 1 has gradually been increased. In Fig. 2 can be observed that the fundamental losses in phase A decrease for the first two cases, but when the injected power is further increased the losses tend to increase. The amplitude of the current that flows in the opposite direction as in the base case is now larger than in the base case. The harmonic losses, however, always increase with larger injected power. The losses in the neutral conductor of feeder **I** become significant due to the asymmetrical input of power. They take rise to 24.09% of the losses in feeder **I** for case 4. When the losses in the entire network are considered, the losses decrease for higher injected power (Fig. 5), also for case 4 where the losses increased in Feeder **I**. This is caused by the lower losses in the supply feeder as can be deduced from Fig. 4. The reduction of the total losses compared to the base case run up to 5.89% for case 2 to 12.87% in case 3 and 13.35% for case 4. The harmonic losses add up to 12.38% of the total losses in case 2 till 14.05% in case 4. The losses caused by the unbalance increase from 0.61% in case 2 till 5.27% in case 4.

The cases 5-7 describe the same increase of DG output but the DG unit is now connected on the subfeeder d2 from feeder **III**. The effect is similar to the cases 2-4, viz the fundamental losses start to decrease with higher input powers until a minimum is reached, then the losses start to increase. The harmonic losses increase anyhow with increasing power input. The biggest difference is that the increase in case 7 is higher than the increase observed in case 4. This can be explained by the higher load demand in node 1 as compared to node d2. In this case the evolution of the losses in the neutral conductor is visible and takes rise to 19.26% of the losses in feeder **III** and subfeeders for case 7. The reduction of the total losses compared to the base case run up to 6.21% for case 5 to 12.78% in case 6 and 11.48% for case 7. As opposed to the DG unit connected to node 1, the reduction of the losses of 6kW DG unit is higher than for a 9kW unit. This is because the load demand in d2 is lower. The losses caused by the unbalance increase from 0.47% in case 5 till 5.50% in case 7. The lower percentage in the losses caused by unbalances in case 5 than in the base case, even with lower total losses in case 5, is caused by the lower losses in the neutral conductor, because the losses caused by the asymmetrical load in node 5 are counteracted by the DG unit connected to the same phase.

2) *Influence of location on the feeder*

When case 9 and 10 are compared with case 11 and 12 the influence of the DG placement on a feeder can be investigated. The reduction in case 9 runs up to 6.23% compared to the losses in the base case where the reduction in case 11 amounts to 6.40%. The larger decrease in case 11 can be explained by considering the load demand in the nodes 4 and 6. The load is higher than the injection of the DG unit, so the power flows from the DG unit to the load. In case 11, the specific node lies more downstream than in case 9, so less transport loss is observed in that case. For the cases 10 and 12 the opposite is discerned, namely the reduction of the losses in case 10 gives rise to 14.91% where in case 12 it runs up to 14.76%. In the present cases the injected power in the node is larger than the load demand in the corresponding node, so the contra discours as mentioned above can be applied.

3) *Influence of an unbalanced load distribution*

The consequence of load unbalance combined with an unbalance in the injected power is revealed by considering the cases 11, 13 and 14. The reduction of losses in the cases 13 and 14 run up to 5.10% which is lower than the reduction in case 11 where it amounts to 6.40%. The reason for this lower reduction has two causes. A first cause is the higher losses in the neutral conductor as can be seen in Fig. 3. And a second, more important reason is that connecting a DG unit on a more loaded phase has a larger influence on the losses, that are quadratic with the current, as shown in the following derivation. In the base case the load is unbalanced with a higher consumption in phase A, so the current in that phase I_A is larger than the current in phase B (I_B). To prove the higher loss reduction in case 11 than in case 13 the following condition has to be met (with I_{DG} the current injected by the DG unit).

$$\begin{aligned} RI_A^2 - R(I_A - I_{\mathrm{DG}})^2 &\overset{?}{>} RI_B^2 - R(I_B - I_{\mathrm{DG}})^2 \\ I_A &\overset{?}{>} I_B \end{aligned}$$

This last statement is true because the current in phase A is larger than in phase B as mentioned before.

4) *Validity of superposition principle*

In case 8, the influence of superposition has been tested. This case is a combination of case 2, where the reduction of the total loss amounts to 5.89%, and case 5, where the DG unit gives rise to a reduction of 6.21%. The reduction in case however is not 12.10% but is 11.50%. This can be explained by means of Fig. 4, where can be seen that superposition of the reduction of losses in phase A for the cases 2 and 5 is larger than the reduction in case 8.

From case 15 the same conclusion can be drawn, viz that the superposition principle is not correct. The reduction of the losses in this case run up to 19.28% and not to 24.73% as would be the outcome when the superposition principle would be used.

4. Conclusions

This paper presented the influence of converter-connected distributed generation on the losses in distribution networks. First a brief review of the influence of DG units on the distribution network has been given. A special emphasis was lied on the losses in the distribution network. In order to study the influence of the converter-connected DG units on the losses, a test case has been set up, a simulation method has been presented considering the main drawbacks of an iterative method and the solutions for these instability problems.

The converter-connected DG units give an increase in the harmonic losses, while, until a specific level, the fundamental losses decrease. The combination of the previous conclusions is that the harmonic losses can become significant when converter-connected DG units are used. It can also occur that the losses in the feeder increase with a high amount of injected power but that the losses on the entire network still decrease. When small single-phase DG-units are used, the losses in the neutral can become higher in cases of unbalances. The losses per phase decrease most when the DG-unit is connected to the most loaded phase. As final conclusion one can state that the loss reduction of different cases can not be summed to find the loss reduction for the combined case.

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