Diagnostic information contained in inter-harmonics of a direct and PWM supplied induction machine

Jan Rusek

Chair of Electrical Machines
AGH University of Science and Technology
Al. Mickiewicza 30, 30-059 Krakow (Poland)
phone: +48 126172897, fax: +48 126341096, e-mail: **gerusek@cyf-kr.edu.pl**

Abstract.

The paper contains waveforms of currents for the direct and voltage source inverter supplied induction machine, with account for harmonics and inter-harmonics. By direct supply the grid is assumed to be polluted by 3rd, 5th and 7th harmonics, normally present in real power supply systems. The voltage inverter was assumed to operate in a PWM mode, with perfectly constant DC link current. Spectrums of stator currents for direct supply create reference bases for spectrums by PWM supply.
The model of the machine accounts for dependencies of stator and rotor inductances on rotor angle, resulting from winding and air gap geometry. Calculations of the derivatives of inductances, with respect to rotor angle, allowed for calculation of electromagnetic torque and machine's dynamics, with allowance for speed fluctuation, also in steady state operation.

Key words

direct and PWM supplied induction machine, spectral composition of currents, inter-harmonics

1. Introduction

The aim of the paper is to calculate waveforms of stator phase and rotor bar currents for a direct or voltage source inverter supplied squirrel cage induction machine, with account for both the harmonics and inter-harmonics. By direct supply the grid is assumed to be polluted by 3rd, 5th and 7th harmonics, normally present in a real power supply system. With account to reality, the 1st, 3rd and 7th harmonics are assumed to constitute positive sequences, whereas the 5th the negative one. The voltage inverter is assumed to operate in a PWM mode. Spectrums of stator currents by direct supply are included, to create a reference bases for current spectrums by PWM supply.
Special model of the machine accounts for true dependencies of stator and rotor inductances on rotor angle, resulting from winding and air gap geometry. Calculations of true derivatives of inductances, with respect to rotor angle, allowed for calculation of electromagnetic torque and, in consequence, the machine's dynamics, with allowance for speed fluctuation also in steady state operation [1].
By PWM supply, the carrier frequency was 5 kHz, and the modulating one 50 Hz. All calculations refer to a 122kW, 1000V, 2p= 4, NS/NR = 72/56 slots machine. The loading torque had T0= 55Nm component independent of speed plus a component depending on the square of the speed, reaching T2= 800Nm by full speed of 1500 rpm. The inertia was 4.52 + 0.52 kg·m^2. The model parameters were established on the basis of manufacturer's data, including winding scheme.

2. Direct supply, centrically aligned rotor

Fig.1a shows the startup and steady-state current by a directly supplied machine. Its spectral composition is shown in Fig.1b. Maximum values of phase supply voltages were: 814.56V by 50Hz, 0.4V by 150Hz, 2.2V by 250Hz and 0.5V by 350Hz. Voltage phase shifts were 0,-120,+120 degrees (positive sequence) for 50, 150 and 350Hz and 0,+120,-120 degrees (negative sequence) for 250Hz. Inductances were calculated for 840 rotor positions. The integration step was $2{*}10^{-5}$s and the printout step was $4{*}10^{-5}$s.

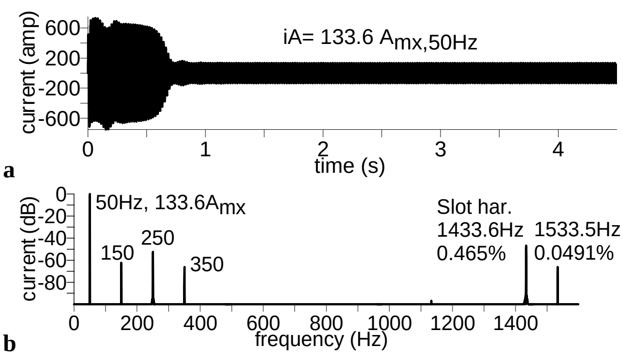

Fig.1. Current and its spectrum, referring to 1.7s - 4.3s segment. h= 1 [2][3]. Frequency resolution Δf= 0.3815Hz.

The presence of the 150, 250 and 350Hz harmonics is the consequence of powering by voltages contaminated by higher harmonics. A slot harmonic frequency of 1433.6Hz follows from the harmonic balance model [2]:

$$f_{Slot} = f_1 + h N_R \, n_{rps} = 50 + 1 \cdot 56 \cdot 24.705 = 1433.48\,Hz \tag{1}$$

where f_1 = 50Hz, parameter h, following from (31) in [2], is h= 1, and the speed n_{rps} = 24.705 rev./s. The discrepancy of 0.12Hz lies within the Δf= 0.3815Hz frequency resolution.
Justification for the presence of harmonic 1533.5Hz in Fig.1b is a bit more complex. It follows from (29) in [2], that the frequency of the parasitic synchronous torque is:

$$f_{TqB} = 2 f_1 + \beta_B n_{rps} = 2 \cdot 50 + 56 \cdot 24.705 = 1483.48\,Hz \tag{2}$$

where the parameter β_B results from (27) in [2]:

$$\beta_B = g \cdot N_R = (4 + 6 \cdot k) \cdot p \tag{3}$$

and, with g= 1, k= 4, amounts to β_B= 56.
This torque harmonic is visible in Fig.2c, and is marked as 1483.5Hz. The discrepancy of 0.1Hz lies within the Δf= 0.3815Hz frequency resolution.

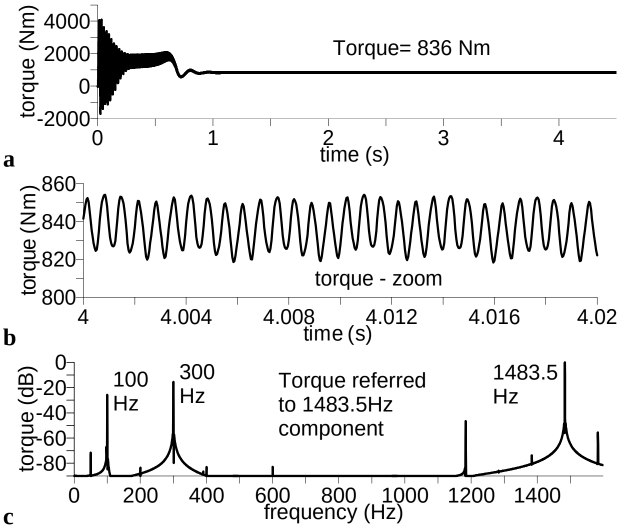

Fig.2. Torque, its zoom and spectrum. Δf= 0.3815Hz.

Torque fluctuations are followed by speed fluctuations of the same frequencies, seen in Fig.3.

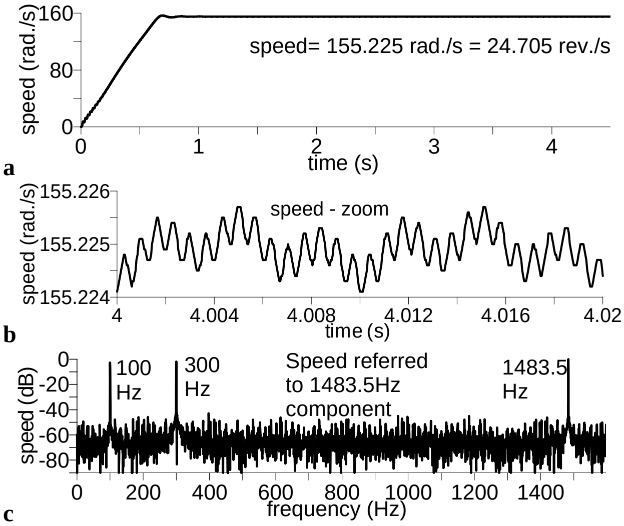

Fig.3. Speed, its zoom and spectrum. Δf= 0.3815Hz.

Speed fluctuation of the frequency of 1483.48Hz would cause modulation of the amplitude of the stator current 50Hz component:

$$i(t) = I_1(1+\cos(2\pi 1483.48))\cdot\cos(2\pi 50) = \\ I_1\cos(2\pi 50) + \tfrac{I_1}{2}\cos(2\pi 1433.48) + \tfrac{I_1}{2}\cos(2\pi 1533.48) \tag{4}$$

Frequency of the penultimate term in (4) coincides with that of slot harmonic. The last term in (4) gives justification for the harmonic 1533.5Hz in Fig.1b. Hence, the reason for the most right harmonic in Fig.1b is speed fluctuation caused by synchronous parasitic torque. Its presence cannot directly follow from harmonic balance model as it refers to constant speed. Let us remind, the current in Fig.1 results from dynamical model, and not from harmonic balance model. The latter one only unanimously justifies the frequency of the slot harmonic, that is that of 1433.6Hz.

3. PWM supply, 40% static + 40% dynamic eccentricity

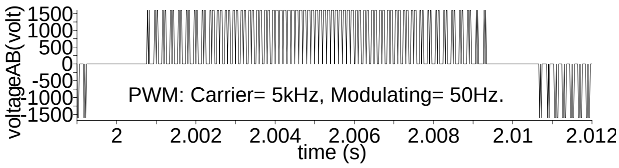

Fig.4 Voltages V_{BC} for the PWM supplied machine with carrier and modulating frequencies f_C= 5kHz and f_M= 50Hz.

Fig.4 shows the current of the PWM powered machine, for the case of mixed eccentricity. The spectrum in Fig.5c shows up harmonics r1L and rL1, characteristic for mixed eccentricity, similar to those present by sinusoidal voltage.

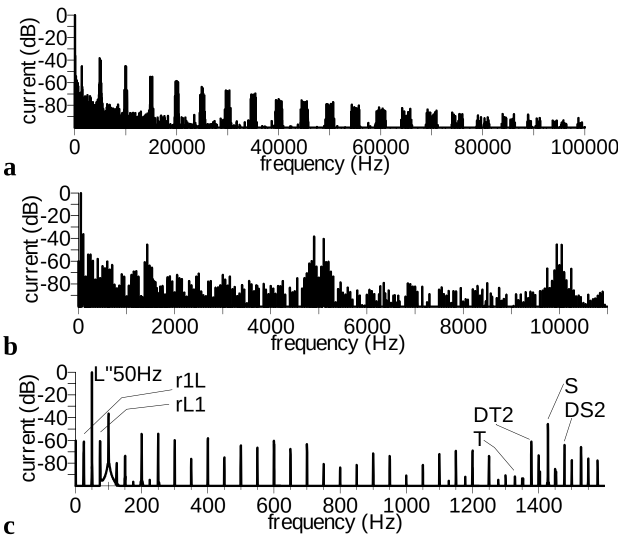

Fig.5. Spectrum of steady-state interval of stator current.

4. Conclusions

1. Rotational harmonics, those around 50Hz, depend multiplicatively on static and dynamic eccentricities. They are only present by mixed eccentricity, that is by simultaneous presence of static and dynamic eccentricity.
2. By PWM supply, distinguishing of the harmonics characteristic to pure static or pure dynamic eccentricity remains still an open issue.
4. Mixed eccentricity rotational harmonics are clearly distinguishable also in case of PWM supply.

References

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Acknowledgement

This work was supported by AGH University of Science and Technology *Statute Work* number 11.11.120.608.