

A Novel Framework for Converter-Driven Stability Assessment in Large-Scale Power Systems

Oriol Cartiel¹, Luis Sainz¹, and Juan José Mesas¹

¹ Department of Electrical Engineering (DEE)
Escola Tècnica Superior d'Enginyeria Industrial de Barcelona (ETSEIB)
Universitat Politècnica de Catalunya - BarcelonaTech (UPC)
Av. Diagonal 647, 08028 Barcelona, Spain.

Abstract. This paper presents a unified framework for converter-driven stability assessment in large-scale power systems. The approach builds on two resonance mode analysis-based criteria: the Lanczos-based positive-mode-damping (L_PMD) and the Arnoldi-based smallest eigenvalue logarithmic derivative (A_SELD). Both methods estimate the smallest eigenvalues of the nodal admittance matrix across a frequency range using sparse techniques, thus enabling efficient identification of oscillatory modes. Their theoretical relationship with state-space eigenvalues is clarified, and their respective strengths and limitations are systematically analysed. Based on this, a combined application strategy is proposed, where L_PMD provides robust oscillatory mode detection and participation factors, while A_SELD enables accurate damping estimation. In addition, parallel computation is implemented to enhance scalability, achieving significant reductions in computational time in large-scale systems. The framework is validated on IEEE and synthetic test power systems, demonstrating accurate stability assessment and substantial improvements in efficiency compared to conventional approaches.

Key words. Large-scale power system, Parallel computation, Positive-mode-damping, Resonance mode analysis, Smallest eigenvalue logarithmic derivative, Stability assessment.

1. Introduction

Converter-driven stability is a growing concern in multi-terminal AC/DC power systems [1]. The two main approaches for stability assessment are time-domain eigenvalue analysis and impedance-based frequency-domain methods [2], [3], [4]. The former evaluates stability by analysing the eigenvalues (or oscillatory modes) $\lambda_k^{ss} = \sigma_k + j\omega_k$ of the state-space matrix $\mathbf{A}$, where σ_k and ω_k denote the damping and oscillation frequency of the system, respectively; instability occurs when $\sigma_k > 0$. Additionally, participation factors (PFs) identify the influence of state variables on oscillatory modes. The latter assess stability using different techniques, including the Generalised Nyquist Criterion, zero-crossings of the aggregated impedance model, the Logarithmic Derivative (LD) of the

aggregated impedance determinant, the LD of the nodal admittance matrix determinant, and Resonance Mode Analysis (RMA) [2], [3], [5], [6]. Particularly, RMA has been extended to stability studies via the Positive-Mode-Damping (PMD) stability criterion [2], which supports black-box models, PFs, and damping margin indicators [4]. However, these methods are computationally demanding for large-scale power systems due to the overload of handling large and sparse nodal admittance matrices $\mathbf{Y}_B$. To address this issue, the highly effective Lanczos-based PMD (L_PMD) [2], [5] and Arnoldi-based Smallest Eigenvalue Logarithmic Derivative (A_SELD) [3] criteria have been proposed as memory- and time-efficient alternatives. These two methods estimate only the smallest eigenvalues of $\mathbf{Y}_B$ using sparse matrix techniques and relating them to the oscillatory modes λ_k^{ss} for stability assessment.

This paper analyses the above RMA-based criteria, clarifies their theoretical foundations, compares their strengths and limitations, and provides a novel framework which highlights their complementary use in practical stability assessment. Additionally, parallel computation (par-com) is proposed to improve efficiency and scalability. The study is validated on IEEE and synthetic test power systems.

2. Resonance Mode Analysis-based Stability Criteria

RMA relies on the eigenvalue decomposition of the $n \times n$ nodal admittance matrix $\mathbf{Y}_{B,\omega} = \mathbf{R}_\omega \mathbf{\Lambda}_{Y,\omega} \mathbf{L}_\omega$ at each frequency $\omega = 2\pi f$ over a frequency range $[f_{ini}, f_{end}]$, where $\mathbf{\Lambda}_{Y,\omega} = \text{diag}\{\lambda_{Y1}, \dots, \lambda_{Yn}\}$ is the $\mathbf{Y}_{B,\omega}$ eigenvalue matrix, and $\mathbf{R}_\omega$ and $\mathbf{L}_\omega$ are the right and left eigenvector matrices, respectively [5]. RMA identifies resonance frequencies $\omega_r = 2\pi f_r$ by detecting peaks in the modal impedance moduli, $|Z_{mi,\omega}| = 1/|\lambda_{Yi,\omega}|$, which correspond to local minima or singularities of $|\lambda_{Yi,\omega}|$. These align with the system's oscillatory modes λ_k^{ss} [2]. Based on this, the resonance

modal impedances $Z_{mi,\omega}$ and modes $\lambda_{Yi,\omega}$ of $\mathbf{Y}_{\mathbf{B},\omega}$ can be expressed as [2], [3]

$$Z_{mi,\omega} = \frac{1}{\lambda_{Yi,\omega}} \quad \lambda_{Yi,\omega} = \kappa \frac{\prod_{q=1}^{n_z} (j\omega - z_{i,q})}{\prod_{q=1}^{n_p} (j\omega - p_{i,q})} \quad (i = 1, \dots, n), \quad (1)$$

where the poles $p_{i,q} = \sigma_{pi,q} \pm j\omega_{pi,q}$ and zeros $z_{i,q} = \sigma_{zi,q} \pm j\omega_{zi,q}$ define the system dynamics, with $z_{i,q}$ coinciding with λ_k^{ss} . The RMA-based PMD stability criterion assesses stability by analysing the impedances $Z_{mi,\omega}$ in (1) [2]. However, this approach is computationally intensive as it requires evaluating the eigentriplets of $\mathbf{Y}_{\mathbf{B},\omega}^{-1}$ across a frequency range. The recently proposed L_PMD [2], [5] and A_SELD [3] stability criteria offer more efficient alternatives.

3. L_PMD and A_SELD Stability Criteria

The L_PMD and A_SELD criteria enhance RMA-based stability analysis for large-scale power systems by computing only the smallest modulus eigenvalue $\lambda_{min,\omega}$ of $\mathbf{Y}_{\mathbf{B},\omega}$ at each frequency using sparse matrix techniques. Consequently, $\lambda_{min,\omega}$ behaves as the piecewise function of $\lambda_{Yi,\omega}$:

$$\lambda_{min,\omega} = \begin{cases} \lambda_{min,\omega}^{(1)} = \left\{ \lambda_{Yi,\omega} / \left| \lambda_{Yi,\omega} \right|_{\min} \right\} & \omega \in [\omega_{ini}, \omega_{min}^{(1)}] \\ \vdots \\ \lambda_{min,\omega}^{(k)} = \left\{ \lambda_{Yi,\omega} / \left| \lambda_{Yi,\omega} \right|_{\min} \right\} & \omega \in [\omega_{min}^{(k-1)}, \omega_{end}] \end{cases} \quad (2)$$

Except at *eigenvalue conflict frequencies*—where segment transitions occur and (2) becomes non-differentiable—(2) follows (1) [3]. Thus, the resonance frequencies of $\lambda_{Yi,\omega}$ and $\lambda_{min,\omega}$, identified as local minima, coincide with the grid's oscillatory modes, $z_i = \lambda_k^{ss}$ in (1). The L_PMD and A_SELD criteria assess stability by linking the frequency scan of $\lambda_{min,\omega}$ to λ_k^{ss} and determining σ_k and oscillatory frequencies ω_k .

A. L_PMD Stability Criterion

The L_PMD stability criterion employs the non-Hermitian Lanczos method [5] to efficiently compute $\lambda_{min,\omega}$ and derive the critical modal impedance $Z_{cm,\omega} = 1/\lambda_{min,\omega}$. Stability is then assessed, assuming that σ_k of oscillatory modes is significantly smaller than ω_k (i.e., $|\sigma_k| \ll \omega_k$ [2]), as follows:

- 1) **Oscillatory frequencies** ω_k are identified at ω_r as the peak values of $|Z_{cm,\omega}|$.
- 2) **The system is stable if and only if**

$$\sigma_k \approx \frac{R_{cm,\omega_r}}{m_{cm,\omega_r}} < 0 \quad \left(m_{cm,\omega_r} = \left. \frac{\partial X_{cm,\omega}}{\partial \omega} \right|_{\omega=\omega_r} \right) \quad \forall \omega_r \approx \omega_k, \quad (3)$$

where R_{cm,ω_r} and X_{cm,ω_r} are the real and imaginary parts of Z_{cm,ω_r} , and m_{cm,ω_r} is the slope of $X_{cm,\omega}$ at ω_r .

- 3) **Left and right eigenvectors** associated with Z_{cm,ω_r} are used to compute the PFs as

$$PF_{b,\omega_r} = R_{b,\omega_r} \cdot L_{b,\omega_r}, \quad (4)$$

where b is the node number. These PFs evaluate the impact of power grid components on stability and are used to calculate a damping margin indicator [4], [5].

B. A_SELD Stability Criterion

The A_SELD stability criterion applies the implicitly restarted Arnoldi method [3] to efficiently compute $\lambda_{min,\omega}$ and determine its logarithmic derivative as

$$D_L(\lambda_{min,\omega}) = \frac{d \ln(\lambda_{min,\omega})}{d(j\omega)} = \frac{1}{j\Delta\omega} \frac{\lambda_{min,\omega+\Delta\omega} - \lambda_{min,\omega}}{\lambda_{min,\omega}}. \quad (5)$$

Then, considering the logarithmic derivative of $\lambda_{Yi,\omega}$,

$$D_L(\lambda_{Yi,\omega}) = \sum_{k=1}^{n_z} \frac{1}{j\omega \pm z_{i,k}} - \sum_{k=1}^{n_p} \frac{1}{j\omega \pm p_{i,k}}, \quad (6)$$

stability is assessed as follows [3]:

- 1) **Oscillatory frequencies** ω_k are identified at ω_r as the peak values of the $D_L(\lambda_{min,\omega})$ real part, $\text{Re}\{D_L(\lambda_{min,\omega})\}$.
- 2) **Oscillatory modes** $z_{i,q} = \lambda_k^{ss}$ are identified from the negative slope of the $D_L(\lambda_{min,\omega})$ imaginary part at ω_r , $\text{Im}\{D_L(\lambda_{min,\omega})\}$.
- 3) **The system is stable if and only if**

$$\text{Re}\{D_L(\lambda_{min,\omega})\} > 0 \quad \forall \omega_r, \quad (7)$$

which implies that $\sigma_k \approx -1/\text{Re}\{D_L(\lambda_{min,\omega_r})\} < 0$.

C. Comparison of the Criteria and Novel Framework for Stability Assessment

Table I shows that both L_PMD and A_SELD provide efficient stability assessment but with different limitations. The L_PMD criterion assumes $|\sigma_k| \ll \omega_k$, while, according to (6), A_SELD may relax this condition. However, A_SELD cannot compute PFs since the Arnoldi method does not obtain left eigenvectors [5], and *eigenvalue conflict frequencies* may hinder stability assessment. Despite their limitations, both criteria are powerful stability assessment methods that could be applied simultaneously to maximise their strengths: L_PMD offers clearer visual interpretation and PFs, whereas A_SELD allows precise damping estimation (see Section 5). Based on this synergy, Fig. 1 proposes a flowchart-based novel framework that guides the unified application of both methods.

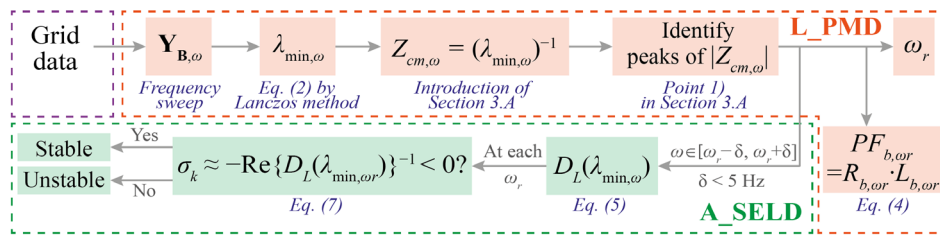


Fig. 1. Novel framework for the unified application of both stability criteria.

Table I. L_PMD & A_SELD strength (★) and limitation (✗) degrees.

	L_PMD criterion	A_SELD criterion
Simple evaluation	★★★	★★★
Low computational effort	★★★	★★★
Oscillatory mode detection	★★★	★★☆
σ_k calculation	★★☆	★★★
Participation factors (PFs)	★★★	✗✗✗
Influence of $ \sigma_k $ value	✗✗✗	✗✗✗
<i>Eigenvalue conflict frequencies</i>	★★★	✗✗✗

4. Parallel Computation for Enhanced Scalability

Both L_PMD and A_SELD stability criteria involve solving an eigenvalue problem at each frequency point within a specified range. Since these frequency-domain computations are independent from one another, par-com can be efficiently applied to accelerate the overall process.

The par-com strategy distributes frequency points among multiple CPU cores (or GPUs), allowing simultaneous eigenvalue calculations. This approach substantially reduces execution time while maintaining accuracy, making it especially suitable for large-scale power systems where thousands of frequency points must be evaluated. The achieved time reduction is approximately proportional to the number of available processing cores.

Efficient parallelisation requires suitable hardware and software configurations. Tools such as the MATLAB Parallel Computing Toolbox facilitate this process by automatically managing parallel task distribution. The RMA-based structure of both criteria provides a natural framework for parallel implementation, as no data dependency exists between frequency-domain computations. Consequently, par-com enhances the scalability of both stability criteria, enabling practical application to high-dimensional test systems such as the IEEE and synthetic grids analysed in Section 5.

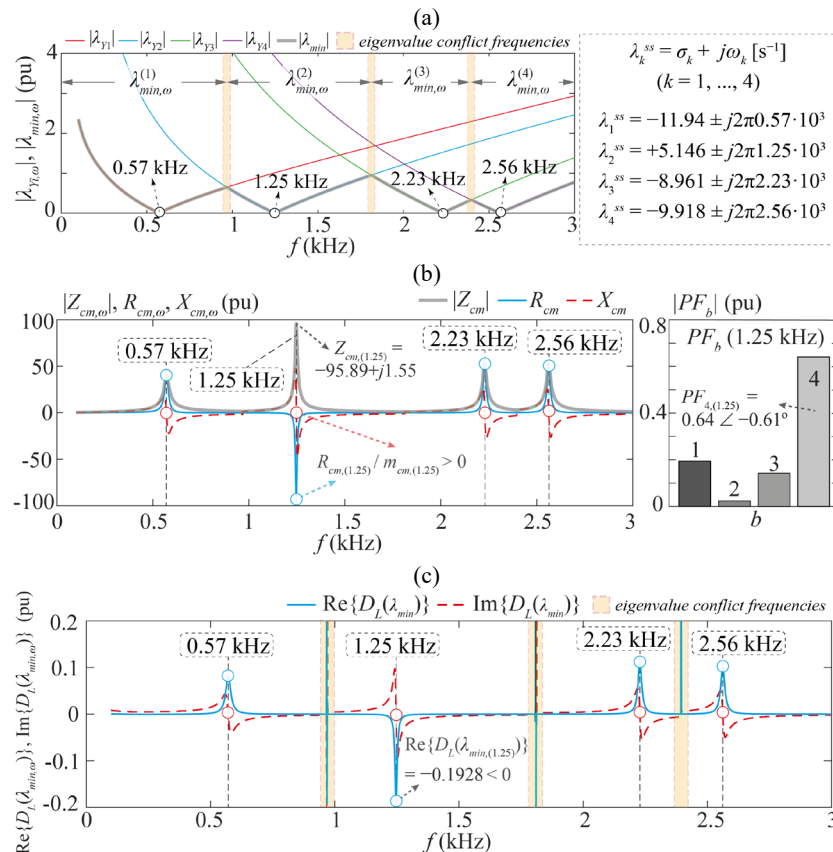


Fig. 2. Stability assessment of the modified IEEE 3-bus system:

(a) Eigenvalue envelope function $|\lambda_{min,\omega}|$ and state-space eigenvalues λ_k^{ss} , (b) L_PMD stability criterion, (c) A_SELD stability criterion.

5. Application

Three case studies are conducted to: (i) validate the accuracy of both criteria by analysing their methodology, verifying the comparison in Section 3, and linking them to state-space matrix eigenvalues; (ii) assess stability in a large-scale power system, demonstrating the scalability and applicability of both criteria and the potential of the novel framework; and (iii) evaluate the improvement in computational efficiency introduced by par-com.

A. Validation of the Stability Criteria

The modified IEEE 3-bus test system in [4], with a VSC system at bus 4, is analysed. Fig. 2(a) shows the minimum-modulus eigenvalue envelope (2) of the four modes. Owing to segment transitions, the function is non-differentiable at *eigenvalue conflict frequencies*. Fig. 2(b) applies L_PMD, identifying four oscillatory frequencies $\omega_r = 2\pi f_r$ as the peaks of $|Z_{cm,\omega}|$. Instability occurs at $f_r = 1.25$ kHz since $\sigma_k \approx R_{cm,\omega_r}/m_{cm,\omega_r} = 5.325 > 0$ (3). The highest PF is PF_{4,ω_r} , indicating that bus 4 (PCC of the VSC) is the most influential on instability. Fig. 2(c) applies A_SELDT, identifying the four oscillatory frequencies from peaks in $\text{Re}\{D_L(\lambda_{min,\omega})\}$, with oscillatory modes derived from the negative slope of $\text{Im}\{D_L(\lambda_{min,\omega_r})\}$. The system's instability at 1.25 kHz is confirmed by $\text{Re}\{D_L(\lambda_{min,\omega_r})\} < 0$, where $\sigma_k \approx -1/\text{Re}\{D_L(\lambda_{min,\omega_r})\} = 5.186$. The state-space matrix eigenvalues λ_k^{ss} are also labelled in Fig. 2(a), showing that their oscillatory frequencies ω_k match the resonance

frequencies ω_r in Fig. 2(b). Additionally, $\sigma_3 = 5.146 > 0$ aligns with L_PMD and A_SELDT results, validating the system's instability due to the oscillatory mode at 1.25 kHz, as predicted by both criteria. Both σ_k and ω_k results confirm the relationship between the state-space and nodal admittance matrix eigenvalues.

B. Scalability Assessment in Large-Scale Power Systems

The modified Texas 2k-bus synthetic test power system [5] is analysed, with four Transformer+VSC systems (using the same parameters as in [4]) at buses 523, 973, 1353, and 1862. Fig. 3(a) presents the L_PMD stability criterion results, identifying multiple oscillatory modes. Despite the system's scale, assessment remains straightforward: detecting ω_r at $|Z_{cm,\omega}|$ peaks and verifying that $R_{cm,\omega_r}/m_{cm,\omega_r} < 0$, which ensures stability (only the sign matters, not the numerical values). The system is unstable due to two oscillatory modes revealed at $f_r = 0.95$ kHz and $f_r = 1.15$ kHz, with the PFs identifying buses 1353 and 1862, respectively, as the most influential owing to the presence of Transformer+VSC PCCs (not shown for brevity). In contrast, Fig. 3(b) illustrates the problem of visualising A_SELDT: *eigenvalue conflict frequencies* create many peaks due to the non-differentiability of (2), obscuring stability-relevant information. This results from multiple segment transitions in large-scale systems. However, zooming into specific frequency ranges enables analysis. Clearly, determining the relevant frequency range without prior L_PMD assessment is significantly

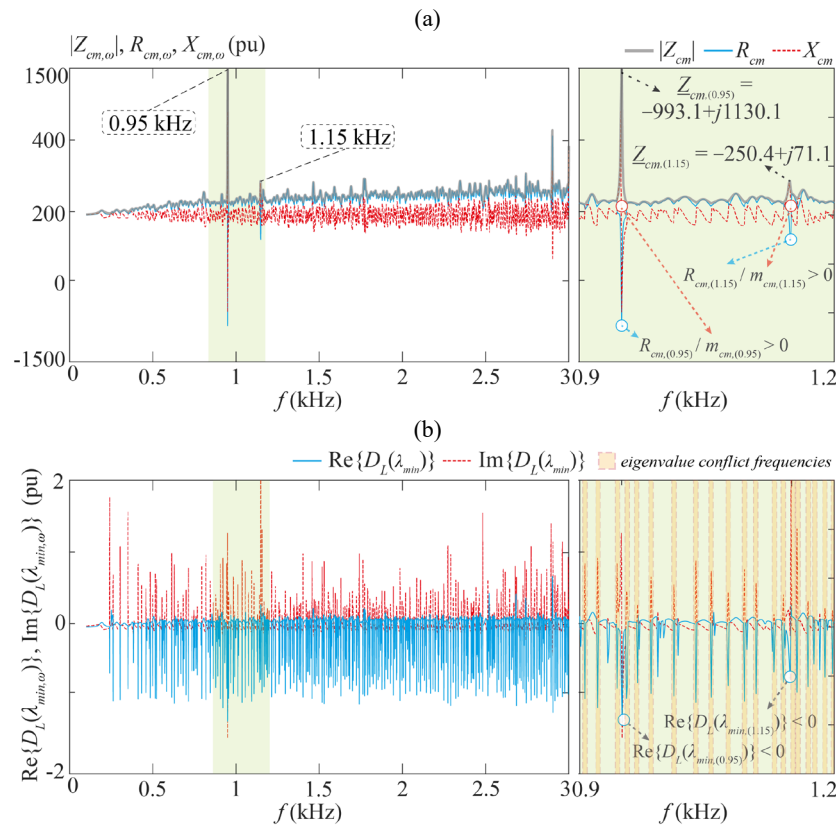


Fig. 3. Stability assessment of the Texas 2k-bus system: (a) L_PMD stability criterion; (b) A_SELDT stability criterion.

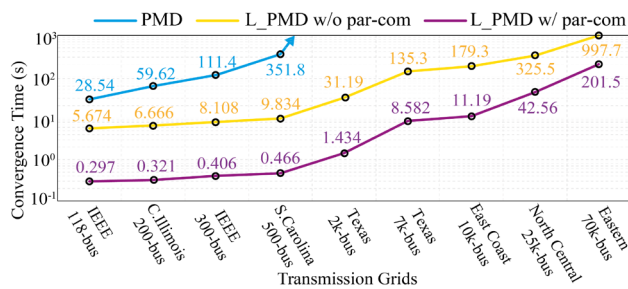


Fig. 4. Computational performance comparison with and without par-com.

more challenging. Nevertheless, A_SELD confirms the same instability issue as L_PMD. Thus, a recommended approach (see Fig. 1) is to use L_PMD for initial frequency identification, followed by A_SELD for refined σ_k analysis, which provides insight into the system's stability degree. Notably, L_PMD recently includes a damping margin indicator, offering a simple conductance/resistance-based measure of system stability [4].

C. Computational Efficiency Analysis

To evaluate par-com impact, a convergence time study is conducted on large-scale IEEE and synthetic test systems [5]. The study evaluates computational performance improvements using a high-performance workstation with an Intel i9-13900K (32 cores), 64 GB RAM, and an RTX 4090 GPU [5]. Fig. 4 compares computational performance in large-scale systems using L_PMD, also including runtimes for traditional RMA-based stability assessment. Results indicate that par-com significantly accelerates convergence. For instance, in the Texas 2k-bus system, computational time without par-com is reduced by a factor of 22 with par-com, whereas in the Eastern USA 70k-bus system, runtime decreases from 16.6 minutes (without par-com) to 3.3 minutes (with par-com). It is verified that A_SELD yields similar results in terms of computational time. Notably, Fig. 4 highlights that traditional RMA-based methods become impractical for large-scale scenarios due to excessive memory requirements and high computational costs [3], [5].

6. Conclusion

This paper establishes a structured comparison between the L_PMD and A_SELD stability criteria, clarifying their theoretical foundations, practical applicability, and relationship to state-space matrix eigenvalues. Rather than revisiting known methods, it consolidates their complementary capabilities into a unified framework for scalable and efficient stability assessment. Additionally, par-com is proposed to reduce computational effort significantly. Its accuracy and scalability are validated on IEEE and synthetic test power systems.

Acknowledgement

This work was supported by Ministerio de Ciencia, Innovación y Universidades, Agencia Estatal de Investigación, FEDER, UE (Grant PID2024-156036OB-C33 supported by MICIU/AEI/10.13039/501100011033 and by FEDER, UE).

References

- [1] N. Hatziargyriou *et al.*, "Definition and Classification of Power System Stability – Revisited & Extended," in *IEEE Transactions on Power Systems*, vol. 36, no. 4, pp. 3271-3281, July 2021.
- [2] L. Orellana, L. Sainz, E. Prieto-Araujo and O. Gomis-Bellmunt, "Stability Assessment for Multi-Feed Grid-Connected VSCs Modeled in the Admittance Matrix Form," in *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 68, no. 9, pp. 3758-3771, Sept. 2021.
- [3] L. Wang, X. Xie, W. Dong, Y. Mei and A. Lei, "Smallest Eigenvalues Based Logarithmic Derivative Method for Computing Dominant Oscillation Modes in Large-Scale Power Systems," in *Journal of Modern Power Systems and Clean Energy*, vol. 13, no. 3, pp. 747-756, May 2025.
- [4] O. Cartiel, L. Sainz, J. J. Mesas and L. Monjo, "A Damping Margin Indicator for Compensator Design by the Positive-Mode-Damping Stability Criterion," in *IEEE Transactions on Power Systems*, vol. 40, no. 4, pp. 3173-3184, July 2025.
- [5] O. Cartiel, J. J. Mesas and L. Sainz, "Efficient Resonance Mode Analysis-Based Methodology for Resonance Studies in Multi-Terminal Transmission Grids," in *IEEE Transactions on Power Delivery*, vol. 40, no. 1, pp. 287-300, Feb. 2025.
- [6] L. Wang, X. Xie, J. Shair and Y. Zhan, "Logarithmic derivative based quantitative analysis of oscillatory stability in renewable powered systems," in *CSEE Journal of Power and Energy Systems*.