

Operating Envelopes Assessment Using Temporally Power Sensitivities Derived from a Data-Driven Approach

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Abstract. The increasing penetration of distributed energy resources poses several challenges in compliance with low-voltage distribution network operating limits. Operating Envelopes describe time-varying export/import limits for distributed energy resources determined to not violate network limits. This paper presents a methodology for estimating dynamic operating envelopes using data from smart meters. Nodal power-voltage sensitivities are estimated using a regression approach to capture the local response under varying operating conditions, and are applied to obtain dynamic nodal operating envelopes without requiring complete network models. The proposed method has been applied to an unbalanced three-phase feeder of 55 loads, demonstrating that nodal voltages are predominantly kept within the maximum allowable limit.

Key words. Operating Envelope, Power Sensitivity coefficients, Smart meters, Overvoltage Limits. Data-driven.

1. Introduction

The increasing penetration of Distributed Energy Resources (DER) in low-voltage electrical grids represents a significant step towards energy sustainability. However, this transition involves challenges that must be mitigated to preserve the safe and reliable operation of the system, such as overvoltages, thermal overloads, and power quality issues. In this context, the concept of operational envelopes (OEs) has been recently proposed as a tool to manage the injection or consumption of active and reactive power by prosumers without exceeding established operating limits.

The OEs represent the maximum power that a distributed generator can inject into the grid over time without violating network limits. Projects such as EDGE [1], Evolve DER [2], and Converge [3] have demonstrated the potential of OE for the safe and efficient operation of electrical grids. The determination and calculation of OEs can be performed using different methodologies. Power flow-based methods are computationally intensive [4], [5], [6], [7]. The variability and uncertainty in the behavior of DERs and demand can be represented with probabilistic models, determining scenarios that allow for the establishment of more robust margins against system variability. Optimization techniques are frequently applied to maximize or minimize an objective function that defines the maximum

OE profiles in a given scenario, subject to network operational constraints [8], [9], [10], [11].

Furthermore, model-free methods have been proposed that derive operating limits from measured power and voltage data using advanced data analysis techniques. The research project "Model-Free Operating Envelopes at NMI Level" [12], [13], [14] presents a neural network-based estimation scheme that leverages smart meter data to calculate the active power OEs of prosumers, considering voltage and thermal constraints. In [15], P-V sensitivity coefficients are estimated from the smart meter data using linear relationships adaptable to different operating states, which are then used in an optimization approach to construct the OEs for each consumer or the secondary substation under a given operating snapshot.

This paper analyzes the estimation of OEs in low-voltage electrical networks, based on active power sensitivities obtained through regression analysis of smart meter data. The method eliminates the need for detailed network models or training processes with complex black-box algorithms such as neural networks. The proposed approach explicitly models temporal variability by estimating data-driven time-sensitivity coefficients, unlike the static approaches analyzed in previous studies. This allows for the direct calculation of dynamic OE values that adjust to the hourly operating state. Furthermore, the proposed approach facilitates the detection of critical nodes and low-capacity time periods for the integration of distributed energy resources (DERs), thus optimizing the operation of the distribution network. The proposed method is applied to the IEEE Low Voltage European test feeder in order to evaluate its validity in preventing voltage problems arising from a high level of DER penetration. The paper is structured in four sections, where Section I is this introduction. In Section 2, the main characteristics of the methodology proposed are outlined. In Section 3 and Section 4, the Study Case and the results obtained are described, and Section 5 is the conclusion.

2. Methodology

The functional relationship between active and reactive power variations and nodal voltage variation can be generally expressed as:

$$\Delta V \approx \frac{\partial V}{\partial P} \Delta P + \frac{\partial V}{\partial Q} \Delta Q \quad (1)$$

where $\partial V/\partial P$ and $\partial V/\partial Q$ represent voltage sensitivity coefficients with respect to changes in active and reactive power, respectively, and ΔP and ΔQ represent changes in nodal active and reactive power.

In low-voltage distribution networks, the R/X ratio is typically high, meaning that voltage variations are primarily driven by active power. Under these conditions, the contribution of reactive power to voltage variations is secondary, especially for small disturbances near the operating point [16], [17], and equation (1) simplifies to:

$$\Delta V \approx \frac{\partial V}{\partial P} \Delta P \quad (2)$$

Based on the above relationship, the power variation can be approximated as a function of the voltage variation by:

$$\Delta P \approx \frac{\partial P}{\partial V} \Delta V \quad (3)$$

where $\partial P/\partial V$ represents the sensitivity of the active power to voltage and allows to obtain the active power that can be injected into a node to not surpass a given voltage level.

This work proposes to obtain this parameter for each node of the network using the nodal voltage and power data by simple linear regression [18] which can be expressed as a least squares problem with the following expression:

$$\beta_{i,h} = (\Delta V_{i,h}^T \Delta V_{i,h})^{-1} \Delta V_{i,h}^T \Delta P_{i,h} \quad (4)$$

where $\beta_{i,h} = \frac{\partial P_{i,h}}{\partial V_{i,h}}$ is the sensitivity coefficient at node i and hour h , and $\Delta V_{i,h}$ $\Delta P_{i,h}$ are the voltage and active power differences between two successive time steps, k and $k-1$:

$$\Delta P_{i,h}(k) = P_{i,h}(k) - P_{i,h}(k-1) \quad (5)$$

$$\Delta V_{i,h}(k) = V_{i,h}(k) - V_{i,h}(k-1) \quad (6)$$

From the calculated values, the OE for node i and hour h can be estimated:

$$OE_{i,h} = \frac{\partial P_{i,h}}{\partial V_{i,h}} * (V^{max} - V_i(k)) \quad (7)$$

where V^{max} is the upper regulatory voltage limit and $V_i(t)$ is the measured voltage at node i at time step k .

Optimized DER injection is defined as:

$$DER_{opt}(t) = \min(DER(t), OE(t)) \quad (8)$$

This formulation ensures that DER generation is only limited when necessary, allowing maximum utilization of available resources and maintaining network security.

3. Study Case

The proposed methodology was applied to the IEEE European Low Voltage test feeder, which represents a 416 V system with 55 single-phase loads distributed across the three phases as shown in Figure 1 [19], [18]. In the study case, loads were modelled using full-year power time series obtained from the SimBench project with a 15-minute time resolution. The dataset was processed using OpenDSS

simulations to calculate voltage values by solving the load flow [20], [21].

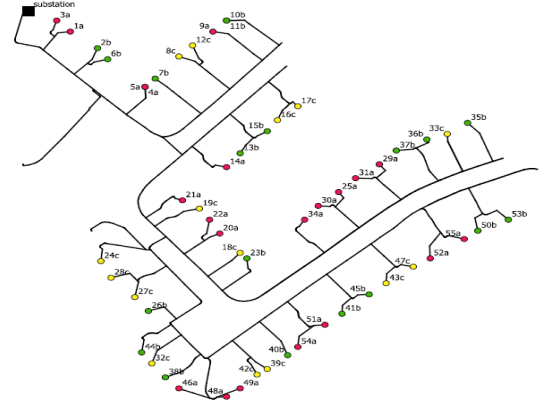


Fig 1. IEEE European LV Test Feeder [19], [18].

Then, the test case was also modified to include photovoltaic DER generation, modelled by using photovoltaic generation profiles from the Zenodo database [22] was added to the consumption nodes of each phase of the test network to transform them into prosumers. These profiles were scaled by a factor of 1.5 to produce network saturation and voltage excursions beyond the established limits. Figure 2 shows the DER injection profiles assigned to each load in phases A, B, and C of the test feeder.

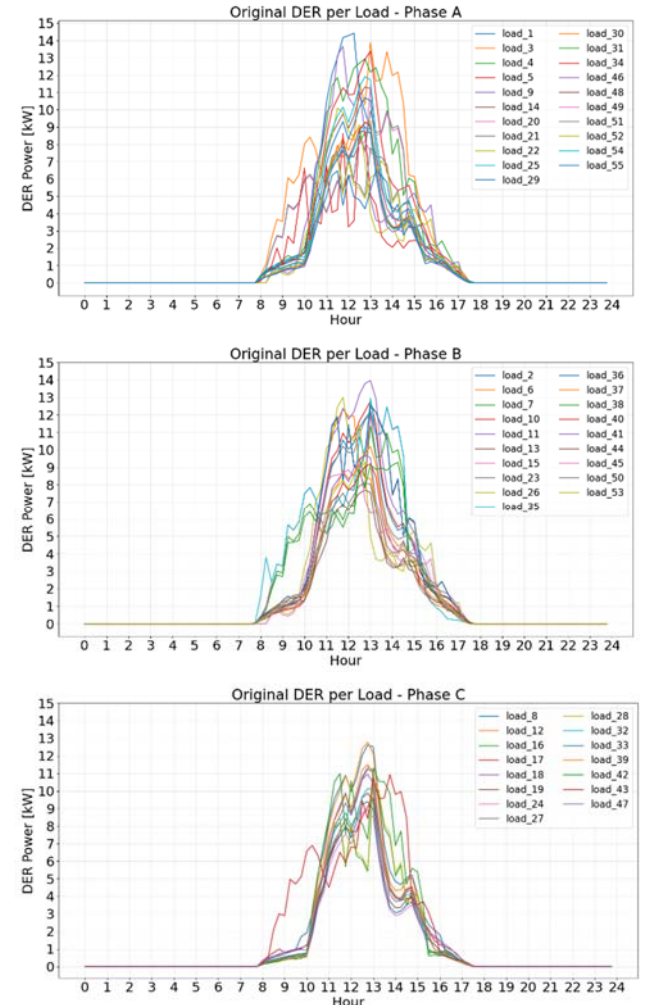


Fig 2. DER profiles assigned per Load – Phases A, B and C

Figure 3 illustrates the resulting nodal voltage profiles obtained after solving the load flow. Several overvoltages are identified at various nodes and time periods, which stem directly from the high penetration of DER.

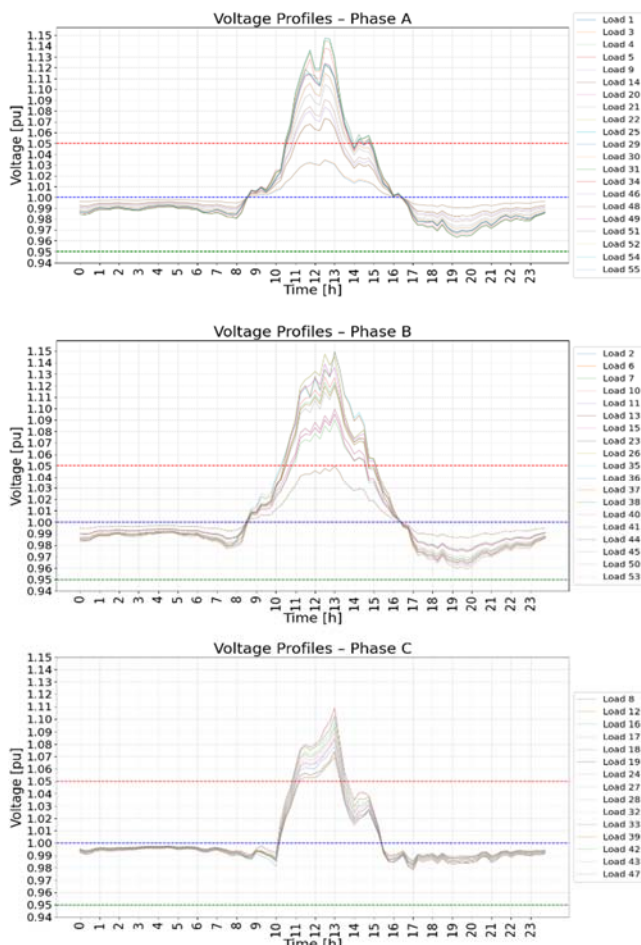


Fig 3. Voltage profile with PV not limited – Phases A, B and C

4. Results

4.1. Sensitivity Coefficients

Hourly sensitivity coefficients were estimated using the full annual dataset, considering 15-minute resolution measurements, which yields four observations per hour.

The sensitivity coefficient for each node i and hour h was estimated by means of equation (4), considering the set of intra-hour measurements corresponding to that hour throughout the year. As a result, hourly sensitivities statistically representative of each time period were obtained.

Figure 4 shows a heat map of the estimated $\partial P/\partial V$ sensitivity coefficients obtained for each node and each hour on the three phases of the feeder. The horizontal axis represents time (24 hours), and the vertical axis represents the nodes, while the color scale indicates the magnitude of the sensitivity in kW/V.

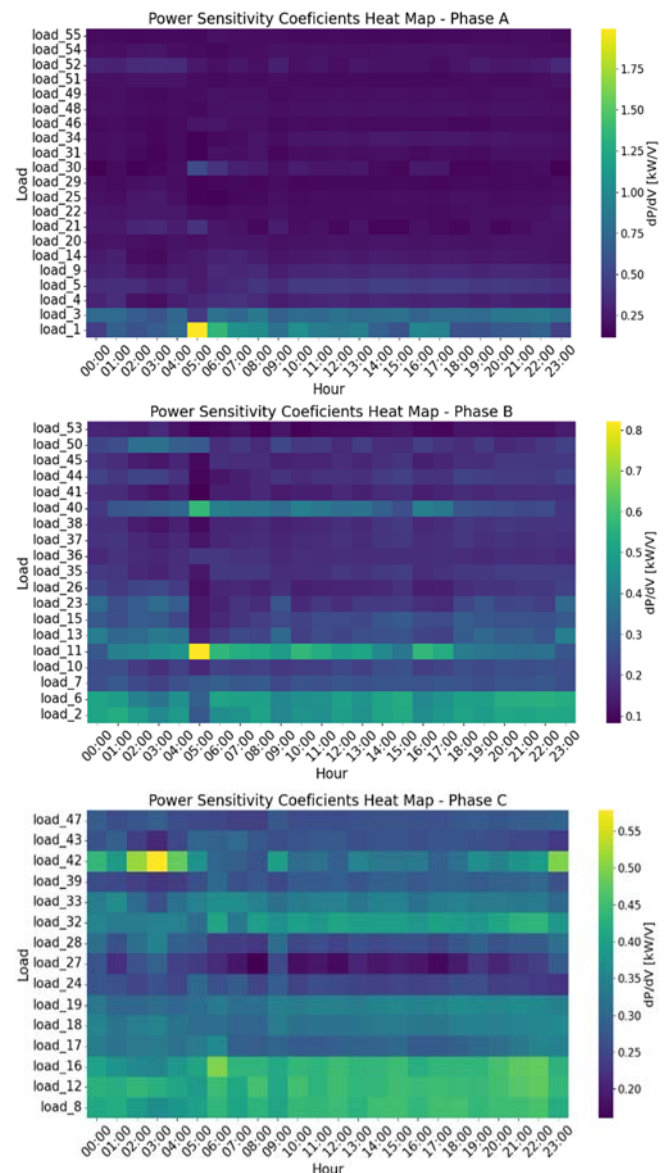


Fig 4. Power sensitivities based on voltage at each hour of the day for all loads per phase.

A marked spatio-temporal heterogeneity is observed. Some nodes exhibit high sensitivities throughout the day (yellow-green), indicating a greater change in power needed to produce a voltage change, typically associated with nodes electrically closer to the transformer or with lower equivalent impedance. In contrast, other nodes show low and relatively stable values (blue-purple), reflecting a lower power-voltage response and, therefore, greater operational constraints. Differences between phases are also evident, attributable to three-phase imbalance and the spatial distribution of loads, confirming that sensitivities depend on both the electrical location of the node and the hourly operating state of the network.

4.2. Operating Envelopes

The OE profiles determine the maximum power injected by a user into the grid while maintaining an acceptable voltage level. This maximum overvoltage level is determined by the applicable regulations. For example, ANSI C84.1 establishes a voltage variation range of $\pm 5\%$.

The calculated coefficients were used to determine the daily OE profiles of every prosumer by using equation (7), under a different load scenario in order to determine the maximum power injection into the node without exceeding voltage limits. Figure 5 presents a box plot of the hourly allowable power defined by the daily OE calculated for the prosumers of each phase.

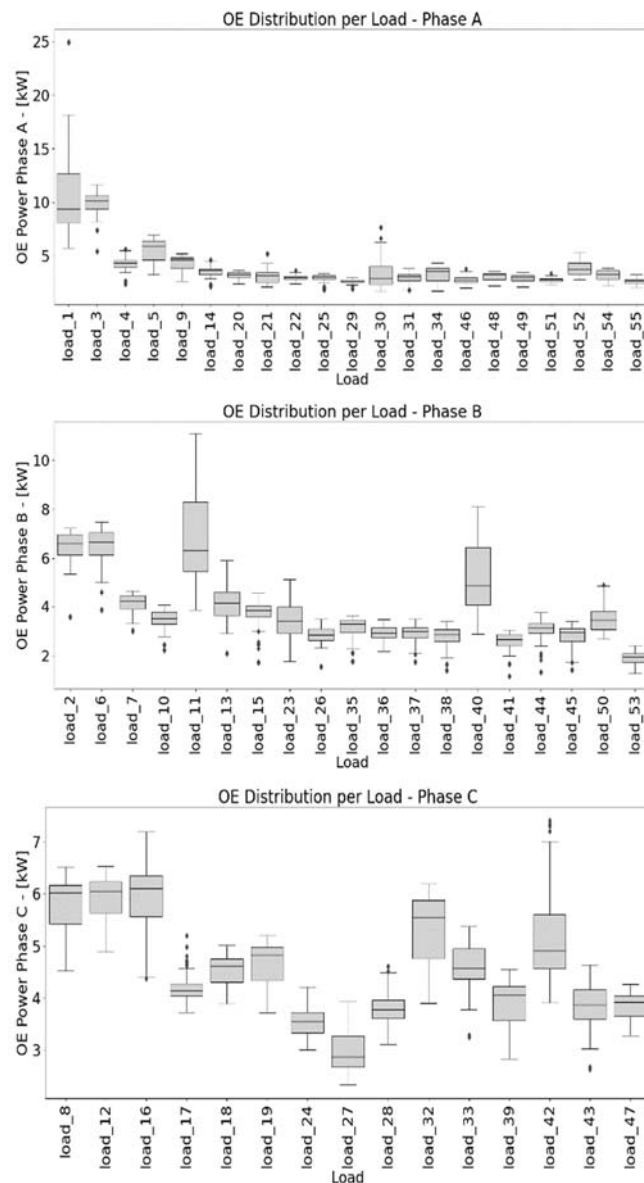


Fig 5. OEs Distribution per Load – Phases A, B, and C

Nodes with high medians and greater dispersion are nodes with greater P-V sensitivity, indicating a larger operating margin but also lower dependence on temporary load conditions. In contrast, remote nodes have more restrictive OEs due to greater accumulated voltage drops and greater sensitivity to power injections.

The consistency of the OEs obtained through nodal sensitivities has been evaluated in the same load scenario, but considering that photovoltaic generation at all nodes is producing the maximum power according to its corresponding OE. The load flow is solved for each hour to determine the voltages and compare their values with the maximum limit, obtaining the results shown in Figure 6,

where box plots with the hourly voltage values per load and phase obtained for the load scenario considered are represented.

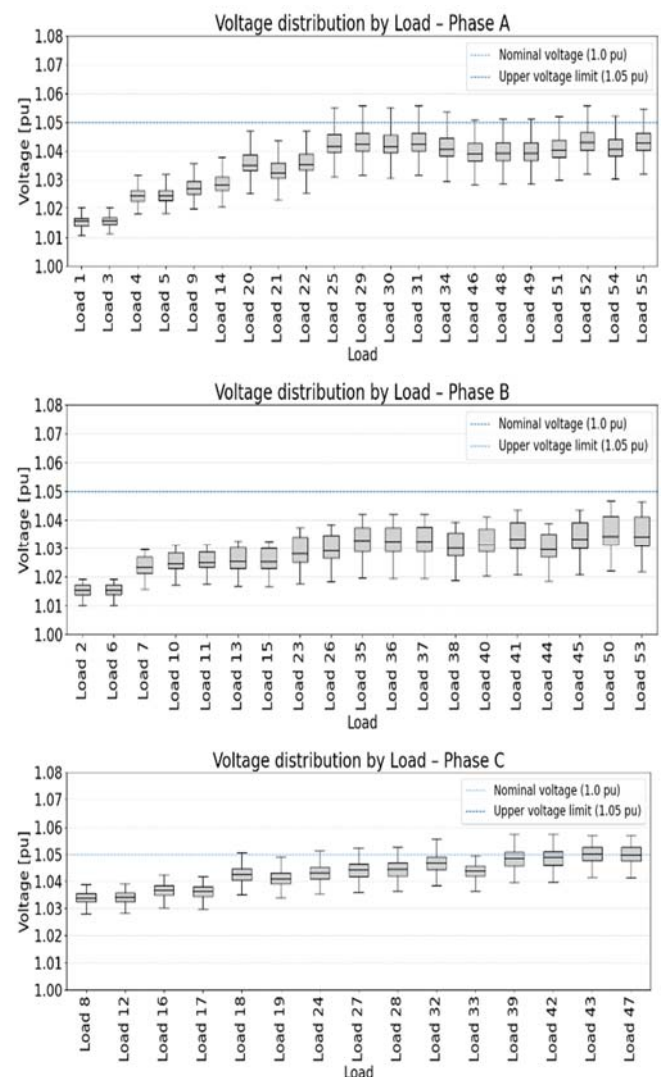


Fig 6. Voltage Distribution with maximum allowable PV as given by OEs – Phases A, B and C

Nodes electrically farther from the transformer exhibit higher voltage levels due to the cumulative effect of distributed photovoltaic injection in the scenario considered, in which all users simultaneously inject their maximum permitted power. The differences observed with respect to the maximum threshold are attributed to the hypotheses adopted in the calculation of sensitivities, which assumes the dependency mainly on the nodal power over other mutual coupling effects. However, the consistency of the resulting profiles validates the robustness of OEs in reproducing operating trends and ensuring system stability under high penetration conditions.

4.3. OEs application in DER allocation

This section analyzes the performance of OEs as a control mechanism to mitigate overvoltages resulting from the injection of power from DERs. In this case, photovoltaic generation profiles assigned to each load node, as

described in Section 3, are now limited when injected power increases beyond the corresponding OE. Figure 7 illustrates the resulting limited DER profiles per load, considering the limits imposed by the OEs calculated for each load in each phase.

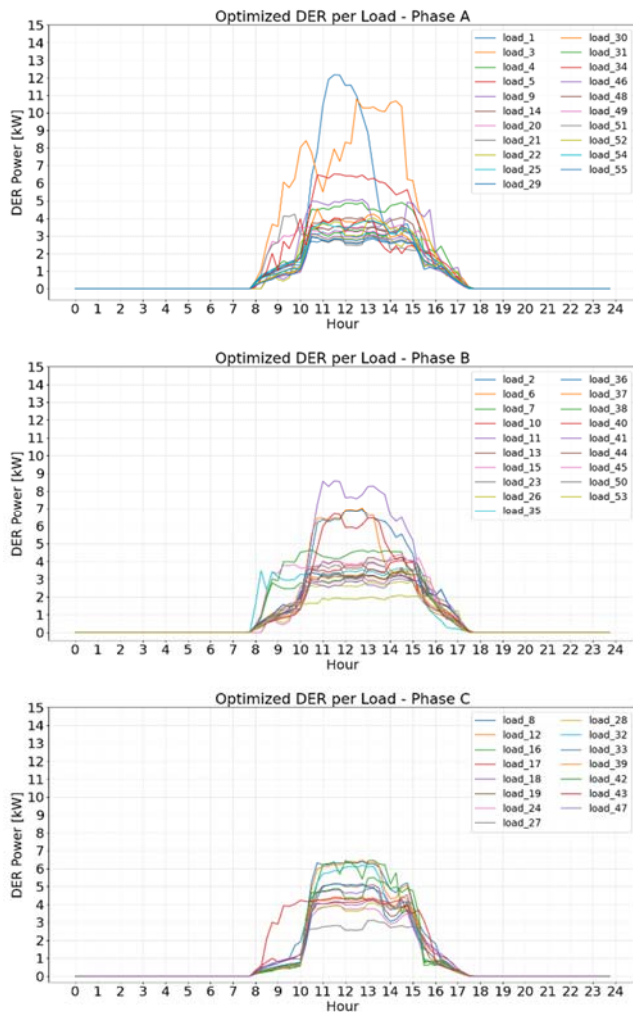


Fig 7. Limitation of PV active power injection by OEs – Phases A, B and C

The graph demonstrates the effectiveness of the OEs in distributing resource allocation, maximizing DER injection without compromising operational safety in terms of voltage. This distribution directly reflects the conclusions drawn from the previously analyzed P-V sensitivities and OEs, confirming that DER integration capacity is highly dependent on nodal location and temporary operating status.

The effect on voltage profiles is presented in Figure 8. This figure shows the nodal voltage profiles for all phase loads obtained as a result of the application of the calculated OEs to limit the active power injection of DERs. As it is shown, the application of these dynamic limits effectively prevents almost all voltage violations ensuring system operation within technical limits, which validates the robustness of the methodology for the management of DER ensuring compliance with voltage restrictions in the distribution network.

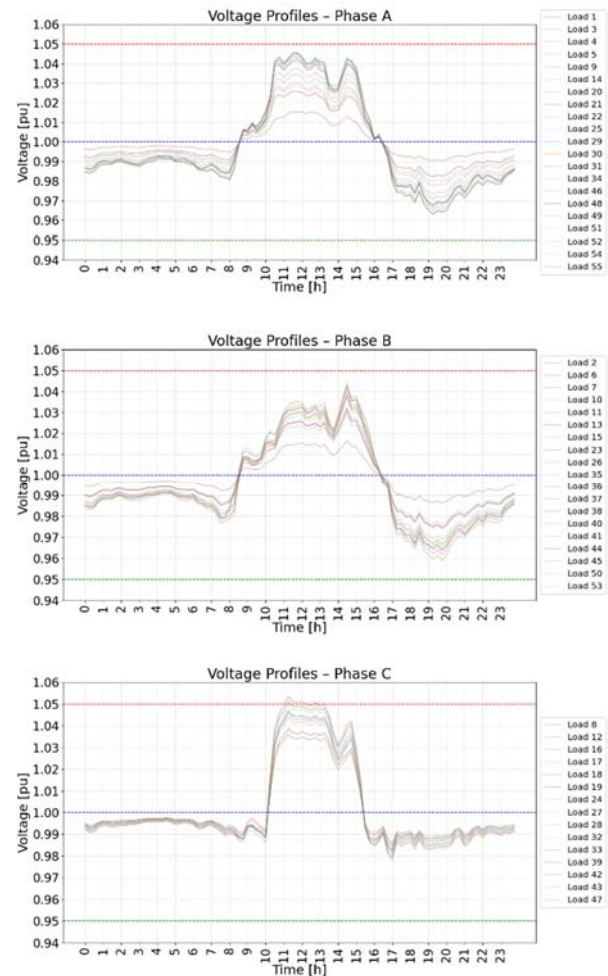


Fig 8. Voltage profile with PV managed by OEs – Phases A, B and C

5. Conclusions

This work demonstrated that time-dependent power-voltage sensitivities constitute an alternative tool for estimating nodal dynamic operating limits from time-series measurements, without requiring a detailed model of the electrical network. The approach allows for capturing the spatio-temporal variability of the network capacity for integrating distributed energy resources, demonstrating that operating limits should not be considered static, but rather a function of the system's time-dependent state. Even in a demanding scenario of simultaneous injection, the methodology reproduces trends consistent with the physical response of the network, supporting its applicability in planning studies and operational analysis.

The results obtained by local linear approximation exhibit small deviations under extreme operating conditions. This suggests the need to incorporate complementary schemes to refine the resulting profiles. Furthermore, the consideration of thermal limitations of critical distribution network components, such as cables and transformers, may also be analyzed, in order to obtain Operating Envelopes profiles that simultaneously reflect the voltage and thermal capabilities of the grid, thus guaranteeing safe network management.

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